

# Associated production of one particle and a Drell-Yan pair

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## Abstract

We briefly discuss the collinear factorization formula for the associated production of one particle and a Drell-Yan pair in hadronic collisions. We outline possible applications of the results to three different research areas.

*Keywords:*

Drell-Yan process, fracture functions, collinear factorization, hard diffraction, underlying event

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## 1. Introduction

The description of particle production in hadronic collisions is interesting and challenging in many aspects. Perturbation theory can be applied whenever a sufficiently hard scale characterizes the scattering process. The comparison of early LHC charged particle spectra with next-to-leading order perturbative QCD predictions [1] shows that the theory offers a rather good description of data at sufficiently high hadronic transverse momentum, of the order of a few GeV. For inelastic scattering processes at even lower transverse momentum, the theoretical description in terms of perturbative QCD breaks down since partonic matrix elements diverge as the transverse momenta of final state parton vanish. The situation is further complicated by the occurrence of additional parton-parton interactions [2] accompanying the main hard process. This effects is possibly ascribed to the composite and extended structure of colliding hadrons. The nature of the latter phenomena, which at present require some kind of modelization, could be more precisely investigated if an improved theoretical description of the main hard process were provided. A well suited process for these kind of studies is represented by the associated production of one particle and a Drell-Yan pair,  $H_1 + H_2 \rightarrow H + \gamma^* + X$ . While the high mass lepton pair constitutes the perturbative trigger which guarantees the applicability of perturbative QCD, the detected hadron  $H$  could then be used, without any phase space restriction, as a local probe to

investigate particle production mechanisms. Within this particular process we will review how to improve the theoretical description to make it capable to handle the aforementioned collinear singularities associated to final state parton production at vanishing transverse momentum. We further note that this process is the single particle counterpart of electroweak-boson plus jets associated production [3], presently calculated at next-to-leading order accuracy with up to three jets in the final state [4]. One virtue of jet requirement is that it indeed avoids the introduction of fragmentation functions to model the final state, which are instead one of the basic ingredients entering our formalism. At variance with our case, however, jet reconstructions at very low transverse momentum starts to be challenging [5] and it makes difficult the study of this interesting portion of the produced particle spectrum.

## 2. Collinear factorization formula

The associated production of a particle and a Drell-Yan pair in term of partonic degrees of freedom starts at  $\mathcal{O}(\alpha_s)$ . One of the contributing diagrams is depicted in Fig. (1). Assuming that the hadronic cross-sections admit a factorization in term of long distance non-perturbative distributions and short distance perturbative calculable matrix elements, predictions based on perturbative QCD are obtained convoluting the relevant partonic sub-process cross-sections,  $d\hat{\sigma}_{ij}^k$ , with parton distribution functions,  $f_i$  and  $f_j$ , and fragmentation functions,  $D^{H/k}$ . The hadronic cross-sections, at center of mass energy squared  $S = (H_1 + H_2)^2$ , can be symbol-

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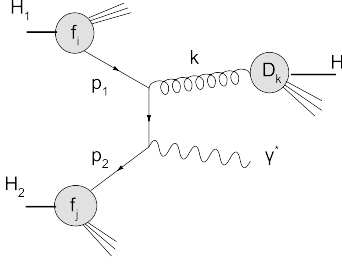


Figure 1: Example of diagram contributing to hadron production in the central fragmentation region to order  $O(\alpha_s)$  in eq.(1).

ically written as [6]

$$\frac{d\sigma^{H,C,(1)}}{dQ^2 dz} \propto \frac{\alpha_s}{2\pi} \sum_{i,j,k} f_i^{[1]} \otimes f_j^{[2]} \otimes D^{H/k} \otimes d\hat{\sigma}_k^{ij}, \quad (1)$$

where the convolution are over the momentum fractions of the incoming and outgoing partons and  $z$  is the energy of the observed hadron  $H$  scaled down by the beam energy,  $\sqrt{S}/2$ , in the hadronic centre of mass system. The invariant mass of the virtual photon is indicated by  $Q^2$ . The partonic indices  $i, j$  and  $k$  in the sum run on the available partonic sub-process. The superscripts label the incoming hadrons and the presence of crossed term is understood. Within this production mechanism, the observed hadron  $H$  is generated by the fragmentation of the final state parton  $k$ , and for this reason we address it as *central*. The amplitudes squared [7], however, are singular when the transverse momentum of the final state parton vanishes. In such configurations, the parent parton  $k$  of the observed hadron  $H$  is collinear the incoming parton  $p_1$  or  $p_2$ . Perturbation theory loses its predictivity as these phase space region are approached. The same pattern of collinear singularities are found also in an analogue calculation in Deep Inelastic Scattering [8]. In both processes such singularities can not be treated with the usual renormalization procedure which amounts to reabsorb collinear divergences into a redefinition of bare parton and fragmentation functions. Such singularities will appear in every fixed order calculation in the same kinematical limits spoiling the convergence of the perturbative series. One possible solution is to regularize the partonic cross-sections with a lower transverse momentum cut-off [9]. It would be, however, highly desirable to develop a technique to resum such logarithmic contributions to all orders in perturbation theory. As shown in Ref. [6], a proper treatment of such singularities requires the introduction of new non-perturbative distributions, fracture functions [10],  $M_i^{H/H_1}(x, z)$ , which parametrize hadron production at vanishing transverse momentum. They

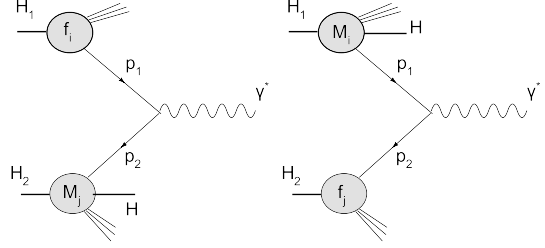


Figure 2: Parton model formula, eq.(2), for the associated production of a particle and a Drell-Yan pair.

express the conditional probability to find a parton  $i$  entering the hard scattering while an hadron  $H$  is produced with fractional momentum  $z$  in the target fragmentation region of the incoming hadron. By using the same generalized collinear subtraction procedure developed for fracture functions in Ref. [8], it is then possible to show that additional singularities discussed above can be properly factorized into the redefinition of bare fracture functions [6]. The fact that the collinear subtraction used in Drell-Yan process is the same as in DIS supports both the universality of collinear singularities among these processes with a different number of hadrons in the initial state and the factorization formula in eq. (2). Fracture functions inhomogeneous evolution equations [10], which can be readily derived starting from this generalized collinear subtraction, can then be used to resum to all orders such type of collinear singularities. The use of fracture functions opens the possibility to have particle production already to  $O(\alpha_s^0)$ , since the hadron  $H$  can be non-perturbatively produced by a fracture function  $M$  itself. The lowest order parton model formula can be symbolically written as

$$\frac{d\sigma^{H,T,(0)}}{dQ^2 dz} \propto \sum_{i,j} [M_i^{[1]} \otimes f_j^{[2]} + M_i^{[2]} \otimes f_j^{[1]}] \quad (2)$$

where the superscripts indicate from which incoming hadron,  $H_1$  or  $H_2$ , the outgoing hadron  $H$  is produced through a fracture functions. This production mechanism is sketched in Fig. (2). So far we have only considered  $O(\alpha_s)$  corrections in the central fragmentation region, eq. (1), to the parton model formula, eq. (2). In order to complete the calculation to  $O(\alpha_s)$  we should also consider higher order corrections in process initiated by a fracture functions. In this case, in fact, the hadron  $H$  is already produced by these distributions and therefore final state parton in real emission diagrams should be integrated over and results added to virtual corrections. One of the contributing diagrams is depicted in Fig. (3). The general structure of these terms is

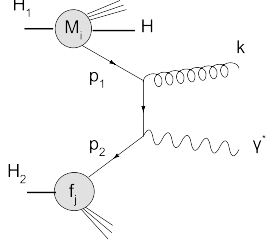


Figure 3: Example of diagram contributing to  $O(\alpha_s)$  corrections in the target fragmentation region, eq.(3).

$$\frac{d\sigma^{H,T,(1)}}{dQ^2 dz} \propto \frac{\alpha_s}{2\pi} \sum_{i,j} [M_i^{[1]} \otimes f_j^{[2]} + M_i^{[2]} \otimes f_j^{[1]}] \otimes d\hat{\sigma}^{ij}(3)$$

We refer to this corrections term as to the *target* fragmentation contribution. The calculation is, a part from different kinematics, completely analogue to the inclusive Drell-Yan case. While soft singularities cancel among real and virtual diagrams, the collinear singularities appearing in this term are renormalized by the homogenous renormalization term in bare fracture functions and by bare parton distributions functions. Adding all the various contributions, the resulting  $p_t$ -integrated cross-sections is then infrared finite [6] and can be sinetically written as

$$\begin{aligned} \frac{d\sigma^H}{dQ^2 dz} \propto \sum_{i,j} [M_i^{[1]} \otimes f_j^{[2]} + (1 \leftrightarrow 2)] \left(1 + \frac{\alpha_s}{2\pi} C^{ij}\right) \\ + \frac{\alpha_s}{2\pi} \sum_{i,j,k} f_i^{[1]} \otimes f_j^{[2]} \otimes D^{H/k} \otimes K_k^{ij}. \end{aligned} \quad (4)$$

We refer to the previous equation as to the collinear factorization formula for the process under study. The next-to-leading coefficients  $C^{ij}$  and  $K_k^{ij}$  have been calculated [11], making the whole calculation ready for numerical implementation. In the following section we will try to outline some applications of the proposed formalism.

### 3. Applications

*a)* The first possible benchmark process for the calculation could be strangeness production associated with a Drell-Yan pair,  $p + p \rightarrow V + \gamma^* + X$ , where  $V$  generically indicates a  $\Lambda^0$  or  $\bar{\Lambda}^0$  hyperon. The first process, according to so called leading particle effect, should be sensitive to the spectator system fragmentation into  $\Lambda^0$  hyperon at very low transverse momentum and therefore to be modelled with the help of fracture functions in  $d\sigma^{H,T}$ . In the anti-hyperon production case no such effect should be present so that  $\bar{\Lambda}^0$  are expected to be

mainly produced by the fragmentation of final state parton as described by  $d\sigma^{H,C}$ . The measurement of such process for different  $Q^2$  could allow to test the fracture functions scale dependence embodied in their peculiar evolutions equations and the validity of the proposed factorization formula in all its components. With this respect, a comparison of strange particle production in hadronic collisions and DIS would be extremely interesting since longitudinal momentum spectra of  $\Lambda$  and  $\bar{\Lambda}^0$  hyperon has been measured quite accurately in charged current neutrino DIS [12], both in the current as well in the target fragmentation region.

*b)* The formalism may found application in the study of single diffractive hard process,  $p + p \rightarrow p + \gamma^* + X$ , where the outgoing proton has almost the incoming proton energy and extremely low transverse momentum with respect to the collision axis. This process has been intensively analyzed in the DIS at HERA, revealing its leading twist nature. From scaling violations of the diffractive structure functions [13] and dijet production in the final state [14, 15] quite precise diffractive parton distributions functions have been extracted from data, which parametrize the parton content of the color singlet exchanged in the  $t$ -channel. The predictions for single diffractive hard processes measured at Tevatron, based on diffractive parton distributions measured at HERA and assuming factorization, has indeed revealed that these processes are significantly suppressed in hadronic collisions, see the very recent analysis reported in Ref. [16]. Recalling that these distributions are fracture functions in the  $z \rightarrow 1$  limit, the present formalism can then applied to next-to-leading order accuracy, the main contribution to the cross-sections coming from the target term,  $d\sigma^{H,T}$ . Such term can be eventually recast in triple differential form in  $x_{IP} \simeq 1 - z$ , virtual photon rapidity,  $y$ , and invariant mass  $Q^2$  and evaluated at next-to-leading order by using the appropriate coefficient functions [17]. In this way factorization tests could be performed at fixed  $x_{IP}$  to avoid any Regge factorization assumption on DPDF while the  $y$  dependence, giving direct access to the fractional parton momentum in the diffractive exchange,  $\beta$ , allows to test factorization in a kinematic region which avoids DPDF extrapolation. Finally, the  $Q^2$  dependence of the cross-sections could be used to investigate how factorization breaking effects eventually evolve with the hardness of the probe and to which extent the factorized formula  $M \otimes f$  actually works.

*c)* The calculation has been performed to make predictions for cross-sections integrated over partonic transverse momentum. To this end, divergent contributions, related to parton emissions at vanishing transverse momentum, are factorized into fracture functions.

In the case, however, that cross-sections are measured down to a minimum but still perturbative hadronic transverse momentum, the latter constitutes a natural infrared regulator for the partonic matrix elements. The central production term,  $d\sigma^{H,C(1)}$ , can be used to estimate hadron production as the fragmentation process, parametrized by fragmentation functions, were happening in the QCD vacuum. The hadronic cross-sections can be recast in a triple differential form in  $Q^2$ , produced hadron transverse momentum  $p_t$  and pseudo-rapidity  $\eta$  to predict charged particle spectra or multiplicity. A particular interesting observable which can also be reconstructed is the differential cross-sections differential in  $\cos\phi$ , where  $\phi$  is the angle formed by the virtual photon and the detected hadron in the center of mass system. This observable has been shown to be sensitive to the contamination of the so-called underlying event [18] to jet observable and has been used also to investigate underlying event properties in Drell-Yan process [19]. Although predictions made on the present formalism take into account the radiation accompanying one single hard scattering per proton-proton interactions, it can be nevertheless used as a reference cross-sections to gauge the impact of new phenomena. The  $d\sigma^{H,C}$  term, although formally  $\mathcal{O}(\alpha_s)$ , is a tree level predictions and possible large higher order corrections may be expected especially in the forward region at large transverse momentum, as already found in DIS analysis [20]. We finally note that the recent analysis of underlying event structure in Drell-Yan events is performed in a wide electroweak boson mass window [19]. We emphasize instead that more insight on the underlying event dynamics could be accessed if cross-sections could be measured, as in the single diffractive case discussed above, at different virtual photon virtualities  $Q^2$  and transverse momentum  $q_t^2$ .

#### 4. Conclusions

We have briefly reviewed a perturbative approach to single particle production associated with a Drell-Yan pair in hadronic collisions. On the theoretical side we have shown that the introduction of new non-perturbative distributions, fracture functions, allows a consistent factorization of new class of collinear singularities stemming for configurations in which the parent parton of the observed hadron is collinear to the incoming parton. The scale dependence induced by this generalized factorization is driven by Altarelli-Parisi inhomogeneous evolution equations for fracture functions which allow the resummation to all orders of this new class of collinear logarithms. The factorization procedure does coincide

with the one used in DIS confirming, as expected, the universal structure of collinear singularities among different hadron initiated processes and supporting the collinear factorization formula proposed in eq. (4). On the phenomenological side we have briefly discussed a few applications in which different aspects of the formalism could be tested and compared towards results already obtained in the DIS target fragmentation region. A good theoretical control on the perturbative component indeed allows the investigation on new phenomena appearing in hadronic collisions, for example the rapidity gap probability suppression in hard diffractive processes with respect to diffractive DIS and the investigation, although indirect, of multiple parton-parton interactions.

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